

Statistical properties of threshold models

Proprietà statistiche dei modelli a soglia

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Riassunto: Il presente lavoro si pone l'obiettivo di mettere a confronto due diverse specificazioni del modello Self Exciting Threshold Autoregressive Moving Average (SETARMA) del quale sono esaminate le condizioni per la stazionarietà. In particolare si evidenzia come utilizzando la specificazione da noi proposta sia possibile definire condizioni per la stazionarietà del modello che indeboliscono alcune restrizioni tradizionalmente poste in letteratura.

Keywords: Threshold models, Stationarity, SETARMA

1. Summary

In the wide class of threshold models, our work focuses the attention on the Self Exciting Threshold Autoregressive Moving Average model (SETARMA). It is able to model data related to phenomena with changes in regimes (such as business cycles, river flows) where the linearity of the generating process cannot be longer sustained.

The aim of the present paper is to analyze and compare two specifications of the SETARMA model. The first is the traditional one introduced in Tong (1983) whereas the second is presented in the following even showing some conditions under which the stationarity of the model is guaranteed.

In particular, given the SETARMA model proposed in Tong (1983)

$$X_t = \phi_0^{(j)} + \sum_{i=1}^{p_j} \phi_i^{(j)} X_{t-i} + a_t^{(j)} - \sum_{i=1}^{q_j} \theta_i^{(j)} a_{t-i}^{(j)} \quad \text{for } X_{t-d} \in R_j, \quad (1)$$

with $\phi_0^{(j)} \in \mathbb{R}$, $a_t^{(j)} \sim WN(0, \sigma_j^2)$ and $R_j \subseteq \mathbb{R}$ such that $R_j \cap R_i = \emptyset$, for $i \neq j = 1, 2, \dots, \ell$, and $\bigcup_{i=1}^{\ell} R_i = \mathbb{R}$. The stationarity of model (1) has been differently examined in Brockwell *et al.* (1992) and Liu and Susko (1992) that fix quite restrictive conditions on the model parameters that can be weakened taking advantage of the theory of the markov processes. In fact, starting from the markovian representation of model (1), in the two regimes case it becomes:

$$X_t = (\delta + \gamma I_{t-d}) + \sum_{j=1}^p \left(\phi_j^{(2)} + \psi_j I_{t-d} \right) X_{t-j} + u_t, \quad (2)$$

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where $I_{t-d} = 1$ if $X_{t-d} \in R_1$ and zero otherwise, $\delta = \phi_0^{(2)}$, $\gamma = \phi_0^{(1)} - \phi_0^{(2)}$, $\psi_j = \phi_j^{(1)} - \phi_j^{(2)}$, with $j = 1, 2, \dots, p$, and $u_t = [\sigma_2 \theta^{(2)}(B) - I_{t-d} (\sigma_2 \theta^{(2)}(B) - \sigma_1 \theta^{(1)}(B))] a_t$. We have shown that model (2) can be even written as:

$$\mathbf{X}_t = \mathbf{1}(\delta + \gamma I_{t-d}) + (\Phi + \Psi I_{t-d})\mathbf{X}_{t-1} + \mathbf{1}u_t,$$

with Φ and Ψ two square matrices of order p properly defined, $\mathbf{X}_t = (X_t, \dots, X_{t-p+1})$, $\mathbf{1}$ a vector of ones and after k iterative steps

$$\begin{aligned} \mathbf{X}_t = & \mathbf{1}(\delta + \gamma I_{t-d}) + \sum_{j=1}^k \left(\prod_{i=0}^{j-1} (\Phi + \Psi I_{t-d-i}) \right) \mathbf{1}(\delta + \gamma I_{t-j-d}) + \mathbf{1}u_t + \\ & + \sum_{j=1}^k \left(\prod_{i=0}^{j-1} (\Phi + \Psi I_{t-d-i}) \right) \mathbf{1}u_{t-j} + \prod_{i=0}^k (\Phi + \Psi I_{t-d-i}) \mathbf{X}_{t-k-1}. \end{aligned} \quad (3)$$

Starting from (3) we have defined sufficient conditions, based on the eigenvalues of $\prod_{i=0}^{j-1} (\Phi + \Psi I_{t-d-i})$, under which the SETARMA model is stationary. They are less restrictive than those fixed in Brockwell *et al.* (1992) and can be further generalized in the ℓ regimes case (with $\ell > 2$).

In particular in this latter case we show that the stationarity of the threshold model can be even reached when the stationarity conditions are not fulfilled by all single regimes involved in the model.

References

- Brockwell P., Liu J. and Tweedie R. (1992) On the existence of stationary threshold autoregressive moving-average processes, *Journal of the Time Series Analysis*, 13, 95–107.
- Liu J. and Susko E. (1992) On strict stationarity and ergodicity of a non-linear arma model, *Journal of Applied Probability*, 29, 363–373.
- Tong H. (1983) *Threshold models in non-linear time series analysis*, Springer-Verlag, Berlin.